

Trends in Grand Unification: unification at strong coupling and composite models*

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We review several problems of conventional Grand Unification and some new approaches. In particular, we discuss strongly coupled Grand Unified Theories. Standard Model may emerge as a low energy effective theory of composite particles in these models. We construct a realistic model of this kind.

1 Conventional unification.

While the Standard Model of elementary particles provides a good description of Nature at energies accessible to current experiments, it is widely believed that the variety of interactions operating in very different ways at our energy scales originate from a single fundamental interaction at high energies. The standard lore (for a review see Ref. [1]) is that (a) the strong, weak, and electromagnetic interactions unify at energy scale $M_{\text{GUT}} \sim 10^{16}$ GeV; (b) the Standard Model is supersymmetric at energies above $M_{\text{SUSY}} \sim 1$ TeV; (c) there are no new particles participating in gauge interactions with masses between M_{SUSY} and M_{GUT} (“the Grand Desert”). This logical possibility is supported by the unification of coupling constants. Running gauge couplings of the Minimal Supersymmetric Standard Model (MSSM) exhibit approximate unification at high energy scale M_{GUT} if their evolution is described by perturbative one- or two-loop renormalization group (RG) equations without extra charged matter heavier than M_{SUSY} . The same is true, though

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parameter-dependent ($\tan \beta$), for some of the Yukawa couplings ($b - \tau$ unification). The assumption of the Grand Desert is important for this conclusion. Supersymmetry is usually introduced because it helps to solve the problem of stability of the Higgs mass in the Standard Model against the radiative corrections. It is also considered as a necessary ingredient of Grand Unification for two main reasons. First, it results in much better unification of couplings. Second, the unification scale is about two orders of magnitude higher than that in theories without supersymmetry. The latter fact is important for better consistency with the observed absence of the proton decay.

However, almost every particular realization of the conventional Grand Unification scenario meets considerable problems. Let us outline briefly a few of them.

Doublet-triplet splitting. In most popular versions of unification, weak $SU(2)$ and colour $SU(3)$ gauge groups are subgroups of a simple group, and all matter fields should fall into complete multiplets of this unification group. As an example, weakly interacting leptons belong to one and the same GUT multiplet as coloured quarks. In a similar way, scalar Higgs doublets ought to have their coloured counterparts. These strongly interacting particles (triplets in the simplest $SU(5)$ model) mediate proton decay at unacceptable rate, unless they are superheavy, i.e. have masses of order M_{GUT} or higher. Unnatural mass difference between particles of one and the same multiplet, doublet and triplet Higgses, can be cured either by fine tuning of parameters or by choosing very complicated Higgs sector.

Potential phenomenological problems. With growing experimental accuracy, it appears that GUTs predict a bit too fast proton decay [2], and also that the coupling constants do not quite merge [3] (unification requires $\alpha_s(M_Z)$ slightly larger than measured).

Supersymmetry breaking. If supersymmetry has anything to do with reality, it has to be broken in the low energy theory. A mechanism that breaks supersymmetry in a phenomenologically acceptable way in the (supersymmetric extensions of the) Standard Model is not known; therefore, another, completely different sector is usually introduced. Supersymmetry breaking in this new, “hidden” or “secluded” sector is often associated with strong interactions due to new gauge fields. These new gauge interactions are not unified with our interactions in conventional GUTs. So, one started from the idea of unification but arrived at a separate gauge interaction in the supersymmetry breaking sector.

Grand Desert. Experimental data and their theoretical analysis point towards the necessity of several mass scales in the Grand Desert. Some of the arguments in favour of the new scales are:

- The most popular solution to the *strong CP problem* requires the axion scale of order 10^{10} GeV (see, e.g., Ref. [4]).
- *Non-vanishing neutrino masses*, in case they are provided by the see-saw or similar mechanism, point towards the mass scale of order $10^{12} - 10^{14}$ GeV (mass of right-handed neutrino) (see, e.g., Ref. [5]).
- *Models of gauge mediation of supersymmetry breaking* make use of the mass scale of messenger fields of order $10^8 - 10^{14}$ GeV (see, e.g., Ref. [6]).

All these arguments suggest that the Grand Desert is actually populated with new particles of various masses.

2 New trends

Is it possible to relax the assumption about the Grand Desert and, at the same time, preserve the self-consistent picture of unification? When a few matter fields are added with masses between M_{SUSY} and M_{GUT} , the gauge coupling unification is preserved at the one-loop level, provided the new states fall into complete representations of a GUT gauge group. However, the two-loop analysis (which is adequate in view of the current experimental accuracy) shows that the unification of couplings becomes considerably worse, as compared to the theory with Grand Desert [7].

On the other hand, with new matter states added, the beta functions change in such a way that the gauge couplings become larger in the ultraviolet. Already with a few additional fields, the asymptotic freedom of QCD is lost, and perturbative RG analysis at high energies may not be applicable. Indeed, the one-loop RG equations for MSSM gauge couplings are

$$\frac{d\alpha_i}{dt} = -b_i\alpha_i^2, \quad (1)$$

where α_i , $i = 1, 2, 3$, denote the gauge couplings of $U(1)$, $SU(2)$ and $SU(3)_C$ gauge groups, respectively, the first coefficients of the beta functions are

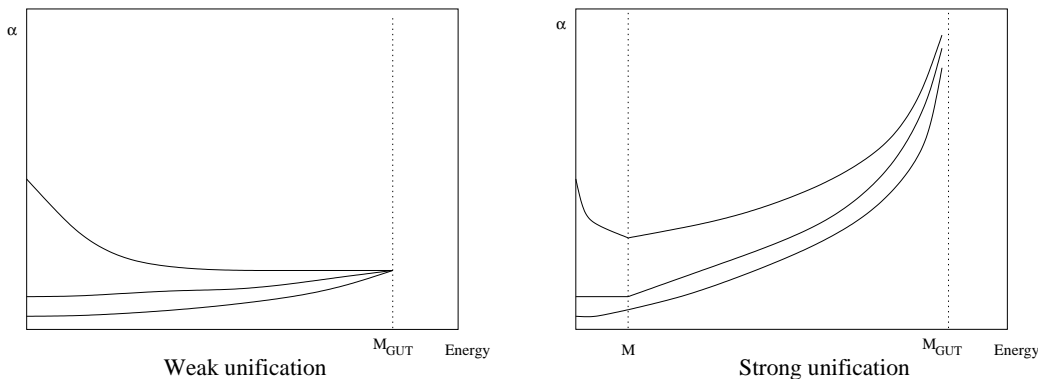


Figure 1: Sketch of running of gauge coupling constants in weak and strong unification scenarios.

$b_1 = -33/5$, $b_2 = -1$, $b_3 = 3$, $t = \frac{1}{2\pi} \ln \frac{Q}{M_{\text{GUT}}}$ and Q is the momentum scale. The solution to Eqs. (1) is

$$\alpha_i^{-1}(Q) = \alpha_i(M_{\text{GUT}})^{-1} + b_i t.$$

For $b_i < 0$, the corresponding couplings grow at high energies. When new matter fields are added, b_i decrease. Suppose that additional particles fall into complete vector-like multiplets of, say, $SU(5)$ unified gauge group, for example, n_5 of $(5 + \bar{5})$ or n_{10} of $(10 + \bar{10})$. Then the coefficients b_i are shifted uniformly,

$$b'_i = b_i - n,$$

where $n = n_5 + 3n_{10}$. For $n > 3$, all three gauge groups are not asymptotically free, and their coupling constants grow in the ultraviolet. For $n \geq 5$, the coupling constants reach their Landau poles below the Planck scale. With measured low energy values of α_i and appropriately chosen masses of additional matter states, however, the three Landau poles coincide (at the one loop, this common Landau pole is always located at $M_{\text{GUT}} \sim 10^{16}$ GeV). This phenomenon is called strong unification (see Fig. 1).

Of course, growing gauge couplings at high energies do not contradict low energy data (in particular, observed asymptotic freedom of QCD) because new matter fields affect RG evolution only at energies of order of their masses and higher.

The possibility that unification may occur in the non-perturbative domain was considered long ago [8] and clearly relaxes the unification constraints because non-perturbative evolution of couplings is unknown. Despite the latter fact, these models are highly predictive because the low energy evolution of gauge couplings is governed by strongly attractive infrared (IR) fixed points [9]. The RG equations (1), written in terms of the *ratios* of coupling constants,

$$\frac{d}{dt} \ln \frac{\alpha_j}{\alpha_i} = b'_i \alpha_i - b'_j \alpha_j,$$

have infrared fixed points,

$$\frac{\alpha_i}{\alpha_j} = \frac{b'_j}{b'_i},$$

which are stable for $b'_i < 0$. In practice, they are so strongly attractive that even at $n = 5$, the ratios are almost constant at $Q < 0.04 M_{\text{GUT}}$. This means that the low energy behaviour of the couplings is independent of the details of their non-perturbative evolution near M_{GUT} , thus keeping the theory predictive.

On the contrary, dynamics at very high energies, of order M_{GUT} and higher, cannot be controlled in the usual way since the theory is strongly coupled at these scales. Various models of high-energy theory can be suggested. Nature may be described at very high energies by a string theory where non-perturbative couplings appear quite naturally. Alternatively, the fundamental theory above M_{GUT} may be some unified gauge theory which is asymptotically free and has confinement scale of order M_{GUT} (another possibility is that this theory is approximately scale-invariant “in the infrared”).

One way to guess the dynamics operating at energies higher than M_{GUT} in strong unification scenario is to consider the Standard Model as a low energy description of a more fundamental strongly coupled theory, in the same way as the sigma model provides the low energy description of QCD. If MSSM is a low energy effective theory, the low energy degrees of freedom, i.e., quarks, leptons, and gauge and Higgs bosons, are composite particles. The compositeness of all these particles used to be problematic because in most models, composite fermions had masses of the order of the compositeness scale (baryons are as heavy as Λ_{QCD}), and no mechanism leading to composite massless gauge bosons was known (in the QCD case, there are no light vector bosons; $m_\rho \sim \Lambda_{\text{QCD}}$). These two properties were major obstacles to construct

composite version of the Standard Model with its gauge bosons and chiral fermions which are almost massless compared to possible compositeness scale (which is not smaller than a few TeV according to current experimental bounds [10]).

Remarkably, both problems are in principle solved by supersymmetry. In the last few years, outstanding progress has been made in understanding the dynamics of strongly coupled supersymmetric gauge theories (see Ref. [11] for reviews). In particular, supersymmetry is so restrictive that sometimes a weakly coupled theory may be uniquely inferred which describes the infrared dynamics of a model with strong coupling at long distances (like in the QCD case). However, unlike QCD, the low energy effective theory of composite particles *is* in a number of cases a theory with massless fermions and massless composite gauge bosons. This phenomenon is known as $N = 1$ duality, and the high- and low-energy theories are often called dual theories. The famous (and first) example [12] is supersymmetric QCD (supersymmetric $SU(N_c)$ gauge theory with N_f flavours of “quark” supermultiplets in the fundamental and antifundamental representations of the gauge group). At $N_c + 1 < N_f < \frac{3}{2}N_c$, the gauge group is strongly coupled in the infrared while the effective low energy theory of composite particles is $SU(N_c - N_f)$ gauge theory with massless matter supermultiplets that carry non-trivial quantum numbers under the low energy gauge group. Many more examples are known (some of them are reviewed in Ref. [11]); their common feature is that the fundamental theory is rather simple but its low energy counterpart may be quite complex and may contain product groups with matter multiplets in chiral representations, very similar to the (supersymmetric version of the) Standard Model.

3 Examples of strongly coupled GUTs

Construction of a realistic composite Standard Model is a non-trivial task; the solution to this problem is not unique and requires guesswork. One possibility is based on the conventional picture of Grand Unification in the weak coupling regime. The Grand Unified Theory itself is then an effective theory of composite particles. An example of a model of this kind was suggested in Ref. [13]. That model is based on the fundamental gauge group $SU(N) \times Sp(4) \times Sp(6)$ with $N \geq 17$ (!); its low energy description is pro-

vided by $SO(10)$ GUT, broken down to the Standard Model by means of the usual Higgs mechanism at weak coupling. Another possibility, which does not require enormous gauge groups of the fundamental theory, is to invoke the idea of strong unification. We present below a class of models with simple fundamental gauge group and MSSM with extra vector-like matter as its low energy description. These models are strongly coupled at GUT scale but, in some sense, represent analytic continuations of usual weakly coupled GUTs.

3.1 The class of models.

Consider an asymptotically free gauge theory with gauge group G and a certain moduli space. At some submanifold of the moduli space, the group G is broken down to its subgroup $G_L \subset G$. Choose matter content in such a way that G_L is IR free. Then the low energy theory at these points of moduli space is described by G_L . It is often possible to add a tree level superpotential that singles out this specific vacuum.

Let the superpotential have a minimum where the expectation values of matter fields are of order v and G is broken down to G_L . Let Λ be the scale at which G is strongly coupled. At $v \gg \Lambda$, the theory is weakly coupled, and the usual Higgs mechanism is in operation. Thus, at energies below v , in particular, in the IR limit, the model is described by G_L gauge theory for any $v \gg \Lambda$. Let us change smoothly the parameters of the superpotential in such a way that v becomes smaller than Λ . Some Green's functions, in particular, those indicating which gauge group remains unbroken, exhibit holomorphic dependence of the parameters. Hence, even at $v \lesssim \Lambda$, the low energy effective theory is again the same G_L gauge theory, free in the IR. This argument enables one to establish the low energy effective description even though at intermediate energies of order Λ the model is strongly coupled, and detailed description of dynamics, in particular, of symmetry breaking, is impossible. We will choose G , matter content, and superpotential in such a way that the low energy theory (G_L) is just the MSSM with extra vector-like matter. Clearly, the model will be an analytical continuation of a usual weakly coupled GUT from $v \gg \Lambda$ down to $v \lesssim \Lambda$.

Very similar arguments appear in the discussion of duality in $SU(n)$ gauge theories with adjoint chiral superfield Φ and a superpotential, Ref. [14]. Without superpotential, the moduli space is described by gauge invariants made of powers of Φ , namely $\text{Tr}\Phi^2, \dots, \text{Tr}\Phi^{n-1}$. At a generic point of the

moduli space, $SU(n)$ is broken down to $U(1)^{n-1}$. There are, however, some special points of extended symmetry where $G_L = SU(k) \times SU(n-k) \times U(1)$. These points remain supersymmetric vacua if the superpotential,

$$W = m\text{Tr}\Phi^2 + \lambda\text{Tr}\Phi^3$$

is added. Consider the case in which, in addition to Φ , there are p fundamental flavours, Q, \bar{Q} . Then the first coefficient of the beta function of G is $b_0^{(n)} = 3n - p$, whereas for subgroups of G_L one has $b_0^{(n-k)} = 3(n-k) - p$, $b_0^{(k)} = 3k - p$. The number p can be adjusted in such a way that G is asymptotically free while G_L is free in the infrared, so the low energy theory is described by G_L . At intermediate energies, the theory is in the conformal phase, and two alternative descriptions are possible — one in terms of (strongly coupled) G_L , another in terms of its dual, $SU(p-k) \times SU(p-n+k) \times U(1)$; both theories are in a strongly coupled phase, and detailed description of the dynamics is not possible.

3.2 An example.

Consider now the implementation of these arguments to the case where $G_L = SU(3) \times SU(2) \times U(1)$. The first coefficient of beta function of G is equal to $b_0^{(G)} = 3l - h - 7 - e$, where $3l$ is the contribution of the gauge superfield, h is the contribution of heavy matter superfields, and $7 + e$ corresponds to light superfields. Here, 7 comes from the MSSM matter (3 chiral generations and electroweak Higgses) and e is the contribution of extra vector-like matter fields. Since G should be asymptotically free, one needs $b_0^{(G)} > 0$. The most restrictive subgroup of G_L is $SU(3)$ for which $b_0^{(SU(3))} = 3 \cdot 3 - 6 - e$ (6 instead of 7 the since Higgs triplets are supposed to be heavy). It is known [9] that for unification at strong coupling below Planck mass it is required that $b_0^{(SU(3))} < -1$. So, one has rather strong restriction on the number of extra matter states,

$$4 < e < 3l - h - 7. \quad (2)$$

For the simplest $SU(5)$ GUT, $l = h = 5$, so the inequality (2) cannot be satisfied. We will discuss $SO(10)$ case in what follows.

The contributions of the lowest $SO(10)$ multiplets to b_0 are:

vector 10	\mapsto	1
spinor 16	\mapsto	2
adjoint 45	\mapsto	8
symmetric tensor 54	\mapsto	12.

$SO(10)$ can be broken down to G_L in two different ways. The most popular way involves Pati–Salam group at the intermediate stage, and requires heavy **54** or higher representations. With $h = 12$, however, the inequality (2) cannot be satisfied. Another way is to embed $G_L \subset SU(5) \subset SO(10)$. This breaking may be achieved by means of (a) heavy adjoint field (see Appendix) or (b) heavy adjoint and **16** + $\overline{\mathbf{16}}$. In the case (a), one needs non-renormalizable superpotential and G_L is not just the MSSM gauge group but contains extra $U(1)'$ factor; one has $4 < e < 9$ in that case. In the case (b), the inequality (2) may be satisfied only if **16** + $\overline{\mathbf{16}}$ can be arranged to be light and contribute to e rather than h .

The theory has two essential mass scales: Λ , where $SO(10)$ becomes strongly coupled, and $v \sim \sqrt{mM}$, where Φ condenses¹ (see Appendix). In the ultraviolet, at energies $\gg \Lambda$, the theory is weakly coupled and correctly described by $SO(10)$. In the infrared, at energies $\ll v$, correct weakly coupled description is provided by G_L (at very low energies, effects of supersymmetry, electroweak symmetry and $U(1)'$ symmetry breaking should be taken into account as well as decoupling of extra matter). If $v \gg \Lambda$, this is a usual weakly coupled GUT; however, at $v \lesssim \Lambda$ the theory is strongly coupled at intermediate energies. In fact, it is expected that the theory is in the conformal phase there, and one of its descriptions is provided (in the case when heavy vector-like matter is in **10**s) by the dual theory of Ref. [13]. The sketch of running of coupling constants is presented in Fig. 2.

Further possibilities emerge in the case of E_6 gauge group where additional vector-like matter states are necessary counterparts of MSSM matter since the lowest E_6 representation, the fundamental, is decomposed under $E_6 \rightarrow SO(10) \times U(1)''$ as **27** \rightarrow **16** + **10** + **1**.

Since the models of the class outlined here are merely analytically continued weakly coupled GUTs, many of usual problems remain unsolved. In particular, the proton lifetime bound imposes usual restrictions on the scale

¹One has to keep in mind that at strong coupling, only gauge invariant degrees of freedom make sense, and vacuum expectation value of Φ , Eq. (3), is to be understood in terms of the expectation values of $\text{Tr}\Phi^2, \dots$. We keep weak-coupling notation for simplicity.

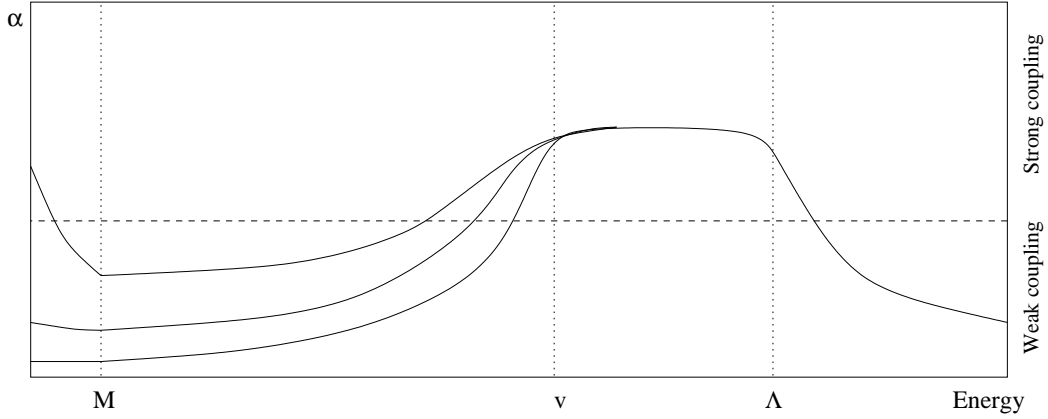


Figure 2: Sketch of running of gauge coupling constants in $SO(10)$ model.

v which is a characteristic mass scale of vector leptoquarks. One might hope that in the conformal phase, usual calculation of proton decay width is invalid since no asymptotic states like leptoquarks are present and the only important scale is Λ (in that case, an interesting possibility of low energy unification would be allowed). However, this is not the case since Green's functions involving baryon number violating operators may still be estimated, and corresponding amplitudes will be suppressed in the usual way by the mass of virtual leptoquarks.

It is assumed, like in usual weakly coupled GUTs, that supersymmetry breaking is provided by different mechanisms. $U(1)'$ is broken if $\mathbf{16} + \overline{\mathbf{16}}$ are involved besides the adjoint; if only adjoint breaks $SO(10)$ (with non-renormalizable superpotential) then $U(1)'$ should also be broken by a different mechanism.

4 Outlook

Strong unification and compositeness may be useful for resolving some of the long-standing difficulties of unification. A low-energy theory of composites does not necessarily contain Higgs triplets since there is no reason to have complete GUT multiplets in the effective theory. Proton could be protected from decay by non-anomalous global symmetries of the theory (which cannot be just baryon number because the latter had to be broken in the early

Universe). The scale of GUT symmetry breaking may be relatively low in this case. At the same scale (which corresponds to the strongly coupled phase), supersymmetry can be broken dynamically in a phenomenologically acceptable way, thus avoiding the hidden sector.

These problems could be solved if the MSSM with extra vector-like matter is a dual theory of a strongly coupled GUT, so that G_L is not a subgroup of G and no fundamental superpotential is involved, like in the case of SQCD [12]. Clearly, much work has to be done before a realistic model is constructed. Phenomenological and cosmological implications of strong unification scenarios of this kind deserve further investigation.

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A Symmetry breaking.

The most general matrix from $so(10)$ algebra is

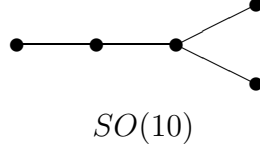
$$\begin{pmatrix} A_1 & B \\ -B^T & A_2 \end{pmatrix}.$$

where A_i , B are 5×5 matrices and $A_{1,2}^T = -A_{1,2}$. If $A_1 = A_2$, $B = B^T$, $\text{Tr} B = 0$, then matrices $(A + iB)$ form the standard $su(5)$ algebra (with anti-hermitian generators). The adjoint field Φ belongs to $so(10)$, too, and its vacuum expectation value with $A_{1,2}^\Phi = 0$, $B^\Phi = \text{diag}(a, a, a, b, b)$ breaks $SO(10)$ down to $G_L = SU(3) \times SU(2) \times U(1) \times U(1)'$ where the Standard Model group is embedded in $SU(5)$ in the usual way, and $U(1)'$ generator has nonzero $\text{Tr} B$, for example,

$$\begin{pmatrix} 0 & 1_{5 \times 5} \\ -1_{5 \times 5} & 0 \end{pmatrix}.$$

Symmetry breaking by adjoint vev is described by the following simple rule [15]. Take the Dynkin diagram for the full symmetry group. Remove one dot

and add $U(1)$ factor instead of it. Different subgroups which may be obtained after breaking with adjoint field correspond to different dots removed. To remove one more dot one needs either the second adjoint or non-renormalizable superpotential. In our case, removing one dot from



results in

$$\left(\begin{array}{c} \bullet \text{---} \bullet \text{---} \bullet \text{---} \bullet \\ SU(5) \end{array} \right) \times U(1) \quad \text{or} \quad \left(\begin{array}{c} \bullet \text{---} \bullet \begin{array}{l} \nearrow \bullet \\ \searrow \bullet \end{array} \\ SO(8) \end{array} \right) \times U(1)$$

or

$$\left(\begin{array}{c} \bullet \quad \bullet \text{---} \bullet \text{---} \bullet \\ SU(2) \times SU(4) \end{array} \right) \times U(1) \quad \text{or} \quad \left(\begin{array}{c} \bullet \text{---} \bullet \quad \bullet \quad \bullet \\ SU(3) \times SU(2) \times SU(2) \end{array} \right) \times U(1).$$

Thus, to obtain

$$\left(\begin{array}{c} \bullet \text{---} \bullet \quad \bullet \\ SU(3) \times SU(2) \end{array} \right) \times U(1) \times U(1)'$$

two dots should be removed. Since introducing the second heavy adjoint contradicts Eq. (2), non-renormalizable superpotential is required. If

$$W = m \text{Tr} \Phi^2 - \frac{1}{M} \text{Tr} \Phi^4,$$

then the equations for supersymmetric minimum written in terms of A^Φ , B^Φ ,

$$\frac{\partial W}{\partial A^\Phi} = \mathcal{O}(A^\Phi),$$

$$\frac{\partial W}{\partial B^\Phi} = \mathcal{O}(A^\Phi) + 4mB^\Phi - \frac{6}{M}(B^\Phi)^3,$$

have a solution with

$$A^\Phi = 0, \quad B^\Phi = \sqrt{\frac{2mM}{3}} \text{diag}(1, 1, 1, -1, -1). \quad (3)$$

This vacuum corresponds to unbroken $G_L = SU(3) \times SU(2) \times U(1) \times U(1)'$.

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